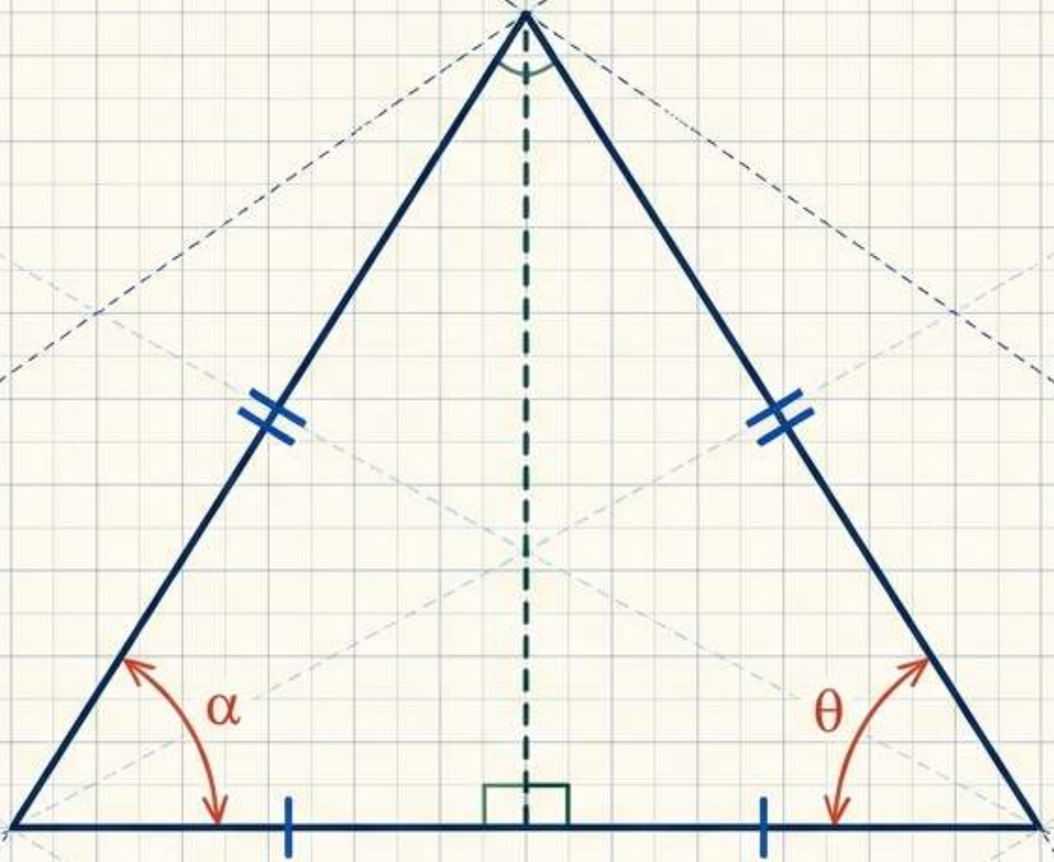
The Laws of the Triangle: Symmetry, Imbalance, and Inequalities

A visual toolkit for decoding the foundational physics of shape.



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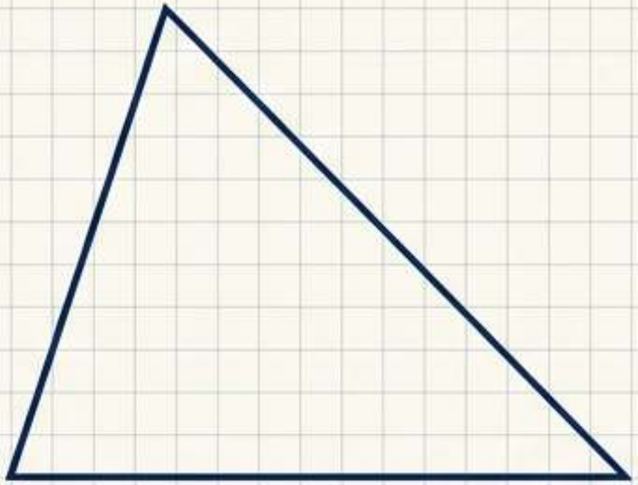
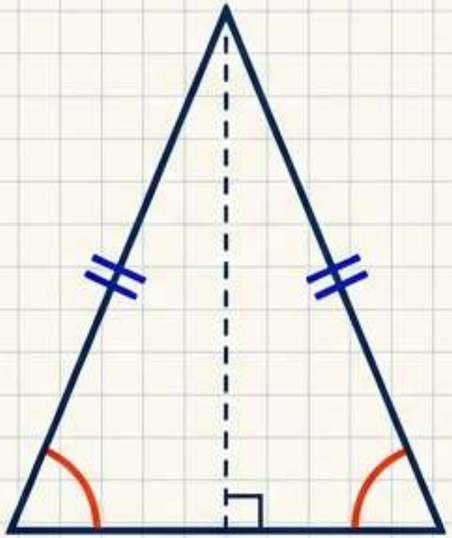
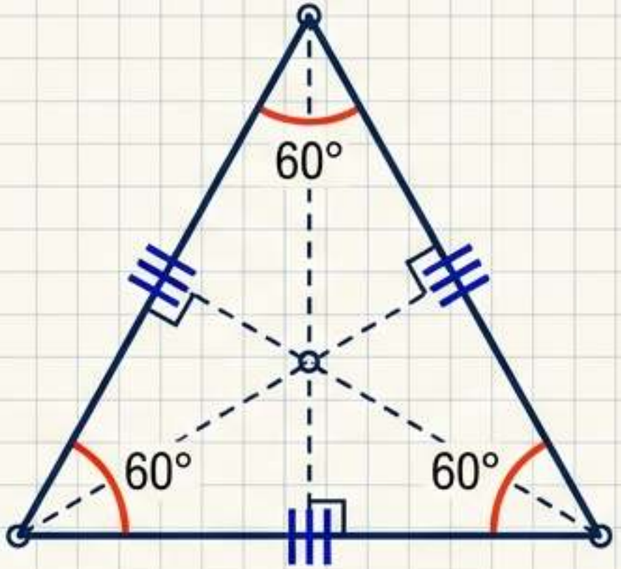
TRIANGLE SPECTRUM ANALYSIS

Scalene	Isosceles	Equilateral
		
Side Equality: None Angle Equality: None	Side Equality: At least two sides equal Angle Equality: Two angles equal	Side Equality: All sides equal Angle Equality: All angles equal (60°)

An equilateral triangle satisfies all the properties of an isosceles triangle—it is a perfected state of symmetry. But an isosceles triangle does not necessarily satisfy the properties of an equilateral.

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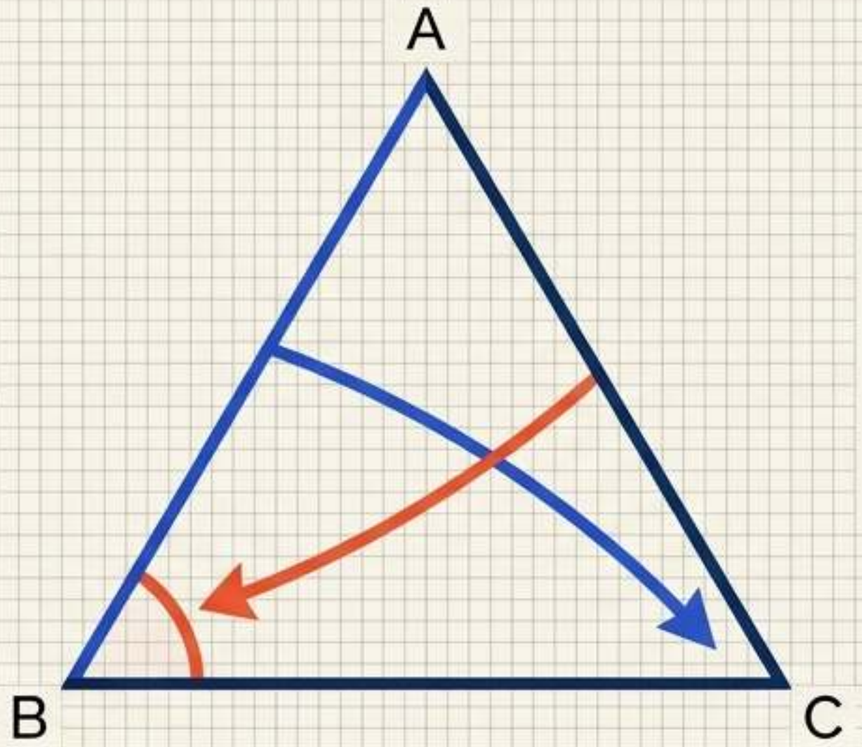
THE SYMMETRY LOOP

Theorem 1: The Forward Law

If two sides are equal (AB = AC), the angles opposite to them are also equal (∠B = ∠C).



Theorem 2: The Reverse Law

If two angles are equal (∠B = ∠C), the sides opposite to them are also equal (AB = AC).

EQUAL SIDES ⇔ EQUAL OPPOSITE ANGLES

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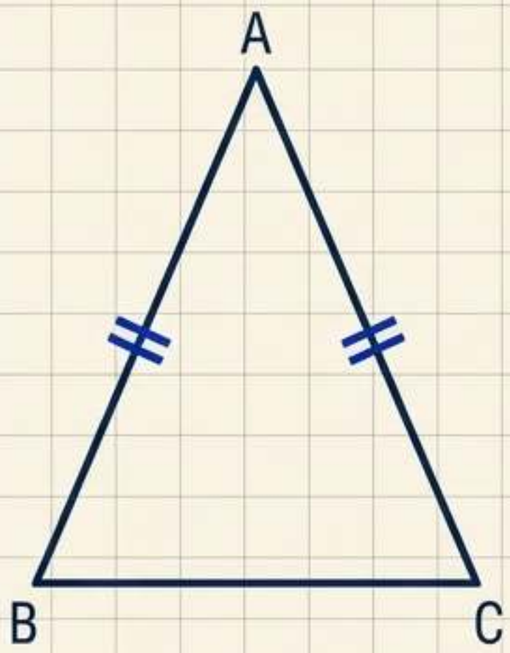
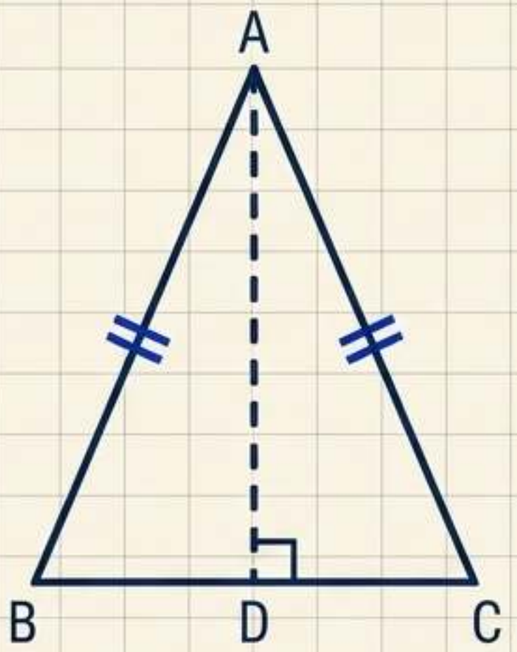
THE RAW SHAPE

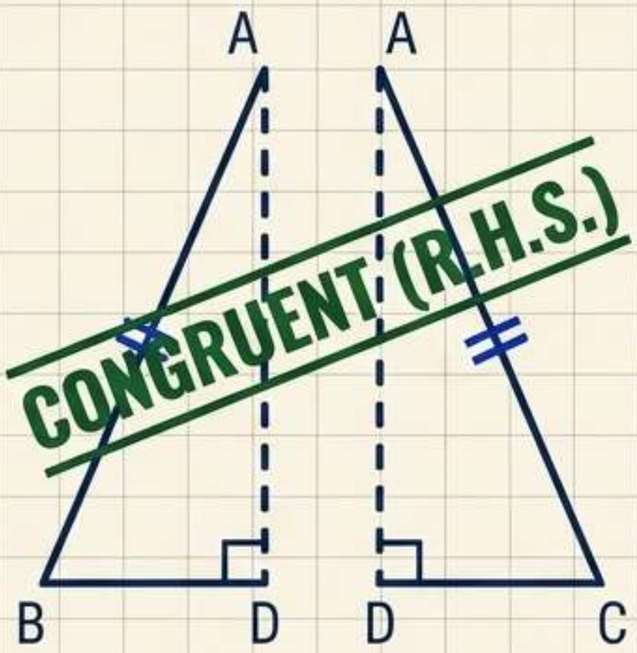


THE CONSTRUCTION



Draw AD perpendicular to BC.

THE MIRROR TEST

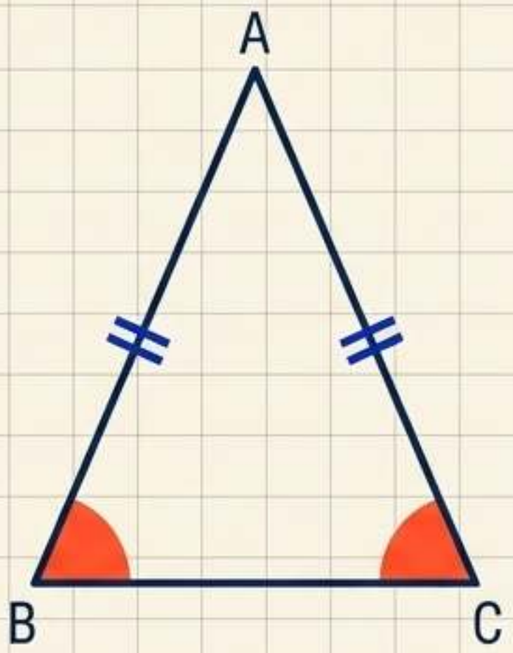


CONGRUENT (R.H.S.)

Compare ΔABD and ΔACD.

- AB = AC (Given)
- AD = AD (Common)
- ∠ADB = ∠ADC (90°)

THE INEVITABLE RESULT



Corresponding parts are equal $\Rightarrow \angle B = \angle C$.

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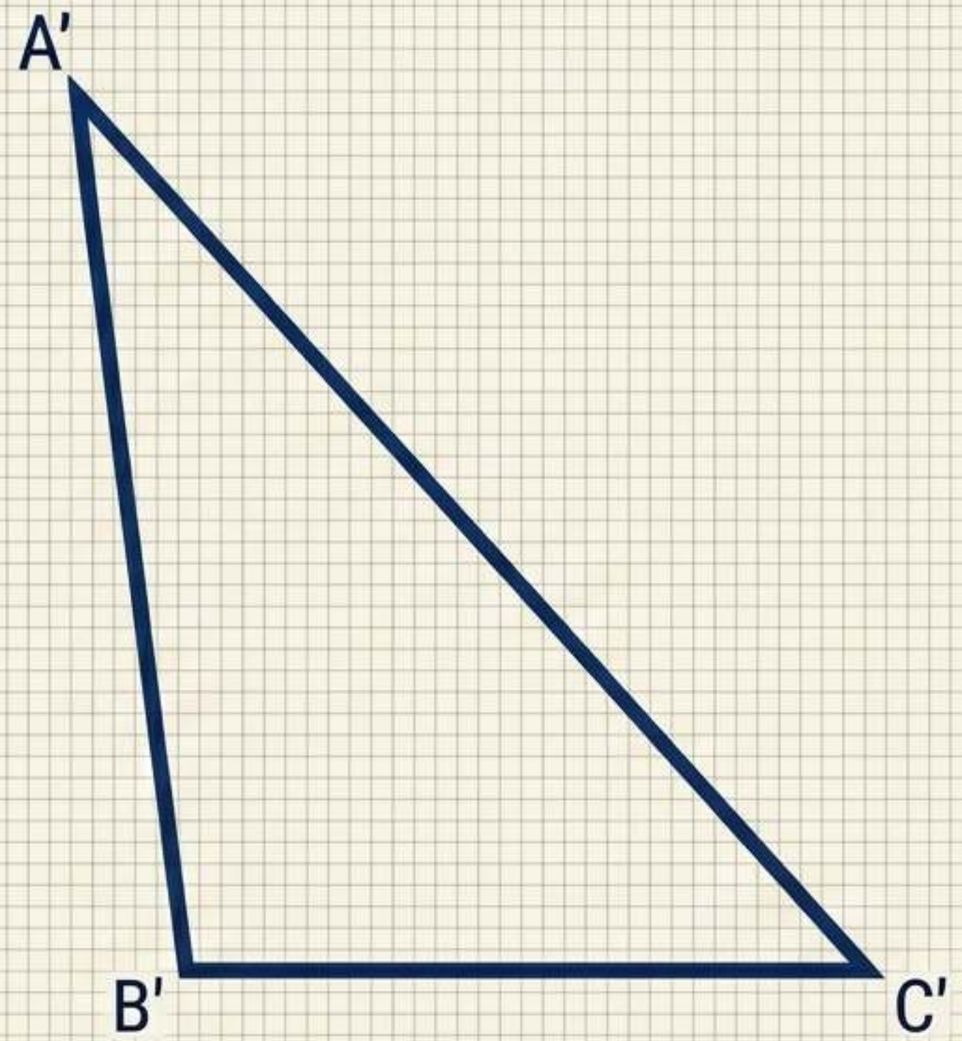
Breaking Symmetry: The Laws of Imbalance

What happens when equality fails? When one side overpowers another, the internal angles must adapt to maintain the structural integrity of the triangle. Welcome to the mathematics of Inequalities.



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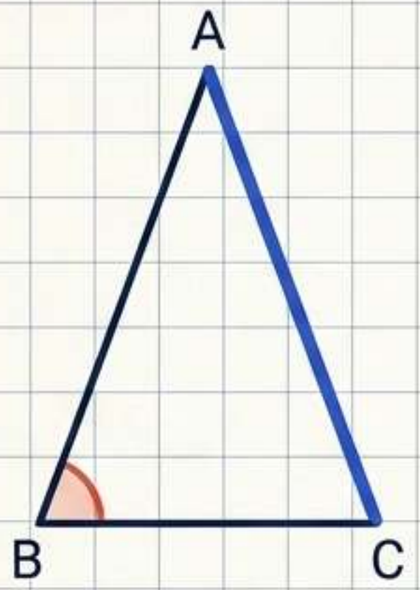
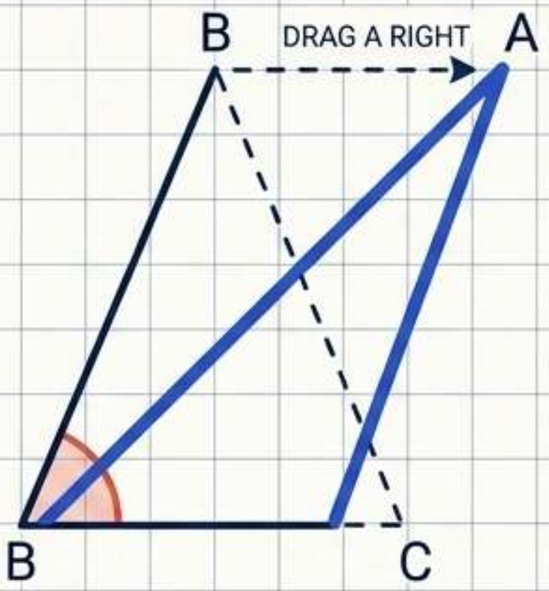
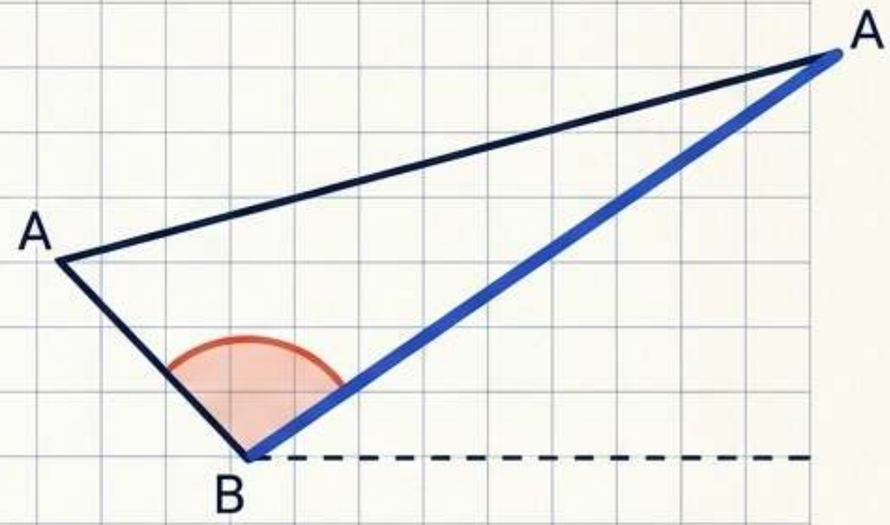
NEARLY ISOSCELES



STRETCHING AC



MASSIVE IMBALANCE



THE LAW OF PROPORTION

If two sides of a triangle are unequal, the longer side commands the greater opposite angle.

$$AB > AC \iff \angle C > \angle B$$

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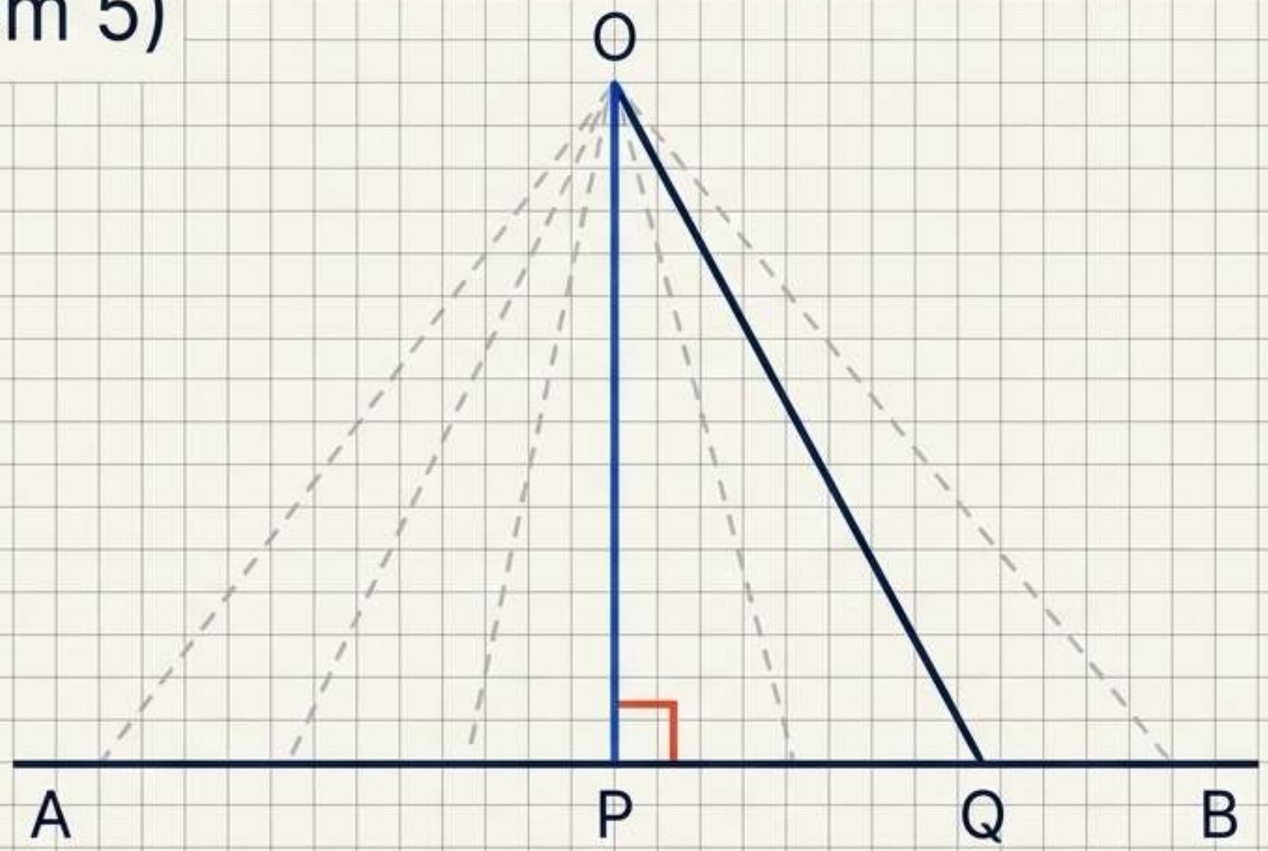
The Shortest Path (Theorem 5)

In right-angled ΔOPQ, the right angle (∠OPQ) is mathematically the greatest possible angle.

Because the longest side must sit opposite the greatest angle (Theorem 4), hypotenuse OQ must be longer than OP.

Therefore, the perpendicular line OP is the shortest distance from any point to a straight line.



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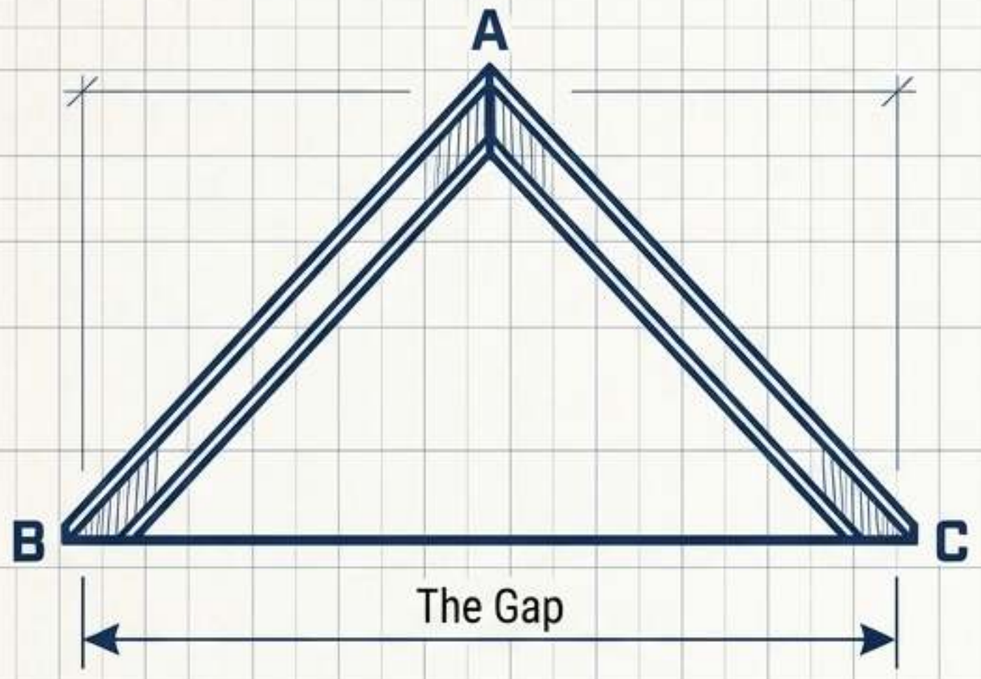
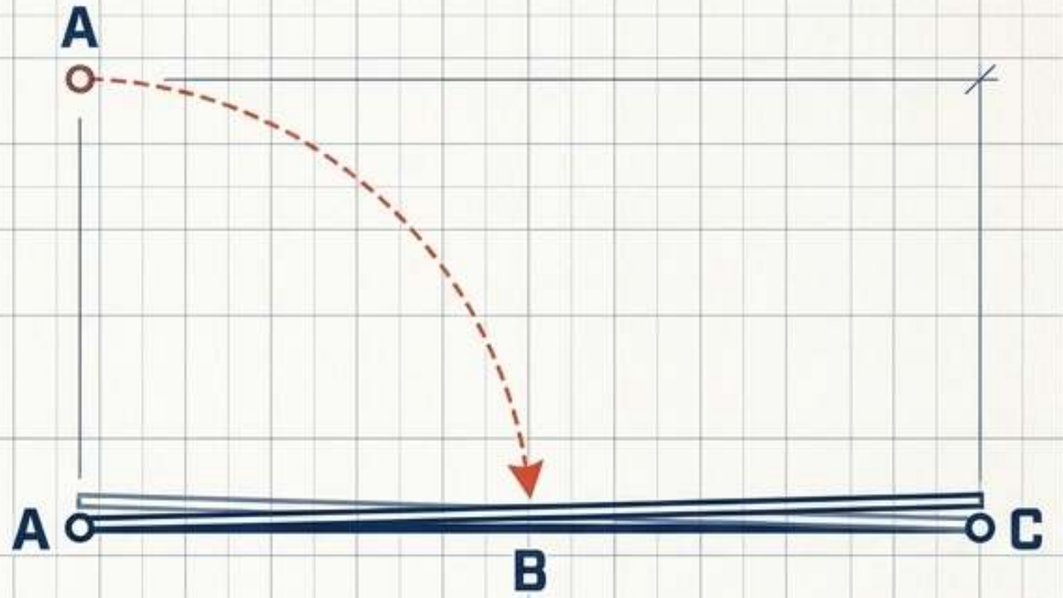
THE CLOSED SHAPE



To span the distance, the sum of the structural arms (AB + AC) must be strictly greater than the base (BC).

THE COLLAPSED BRIDGE



If AB + AC = BC, the shape flattens completely into a straight line. It fails to form a triangle.

EXISTENCE LAW 1: The sum of the lengths of any two sides is always greater than the third side. (e.g., AB + AC > BC)

EXISTENCE LAW 2: The difference between the lengths of any two sides is always less than the third side. (e.g., |AB - AC| < BC)

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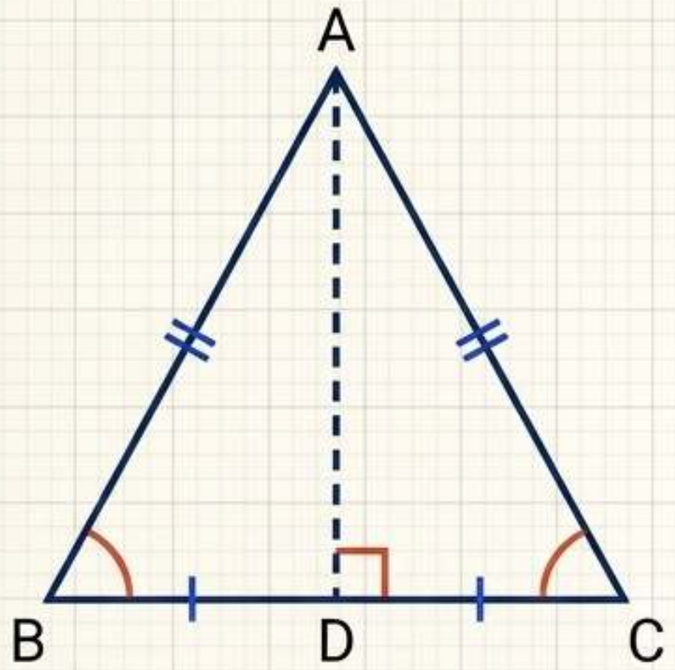
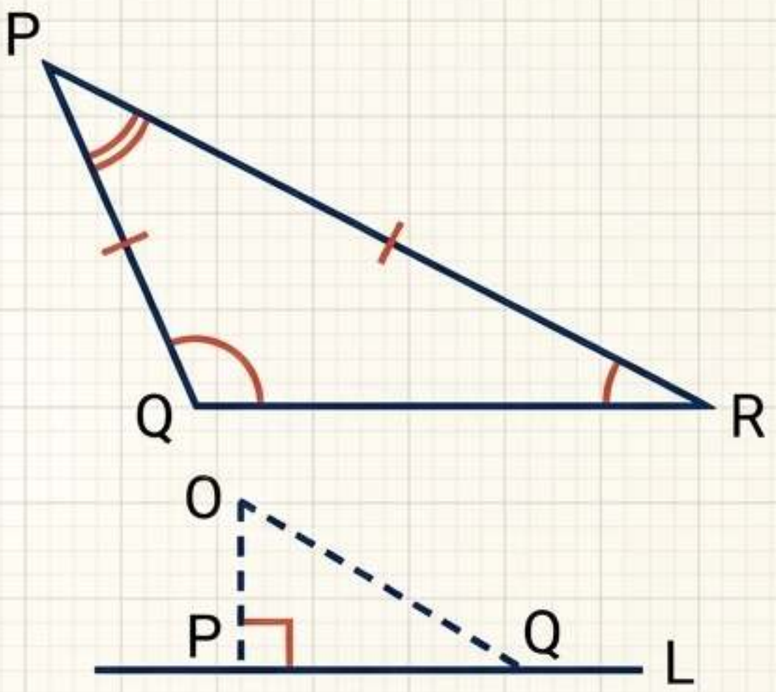
The Geometry Diagnostic Toolkit

The Rules of Symmetry (Isosceles)



- Equal Sides ⟺ Equal Opposite Angles.
- Constructive Tool: Dropping a perpendicular altitude creates two perfectly congruent mirrored halves.

The Rules of Imbalance (Inequalities)



- Longest Side ⟺ Greatest Opposite Angle.
- Existence: Sum of any two sides > Third side.
- Shortest Path = Perpendicular distance.

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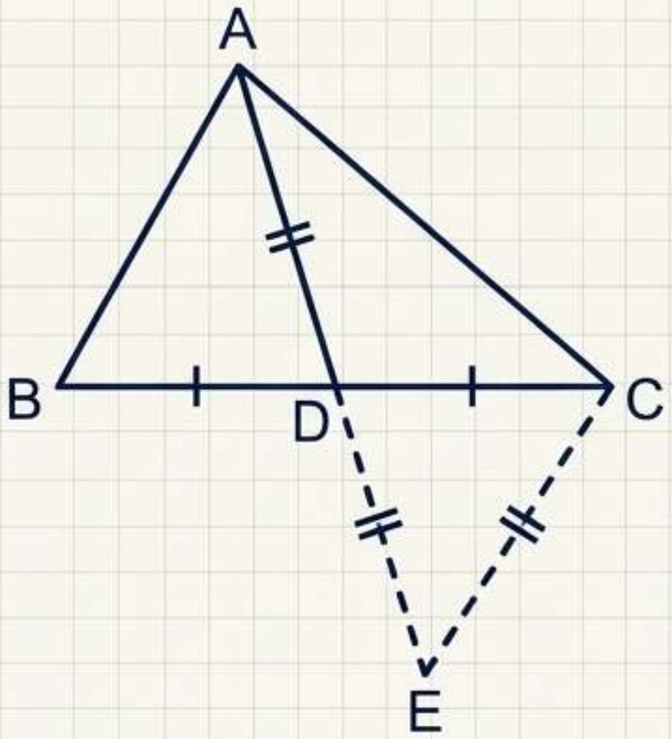
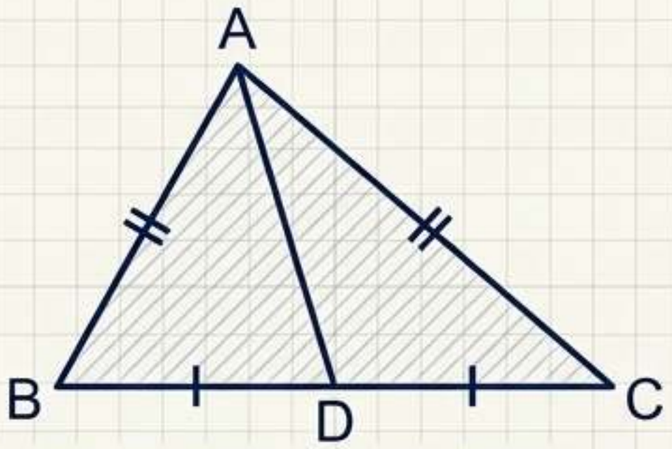
Application & Mastery

Prove that the sum of two sides is greater than twice the median (AB + AC > 2AD)

Step 1: The Construction

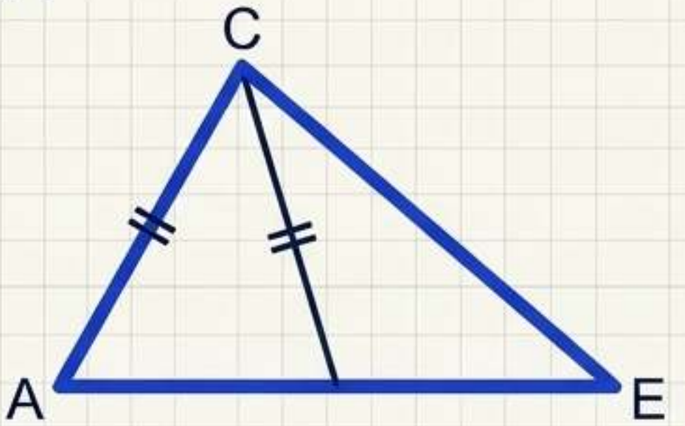


Step 2: The Congruence Flip



By S.A.S. (Side-Angle-Side), these triangles are identical mirrors.
Therefore, AB = CE.

Step 3: The Existence Rule Application



In ΔACE, the sum of two sides must beat the third: AC + CE > AE.

Since CE = AB, and AE = 2AD, substitute them in:

AB + AC > 2AD ✓ **Hence Proved.**

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